Chapter 2: Financial Calculations and Formulas

Ex 2.1: Time Preference for Money

In economics, time preference is the relative valuation placed on a good at an earlier date compared with its valuation at a later date. The time value of money means that the worth of a dollar received today is different from the worth of a dollar to be received in future. For an individual who behaves rationally, a $1 received in future is less valuable than the $1 received today. Moreover, $1 received 2 years from now is far less valuable than the $1 received 1 year from now. This preference for money now, as compared to future money, is called time preference for money.

Time preference is the inclination of a consumer towards current consumption (expenditure) over future consumption, or vice versa.

The time preference for money is generally expressed by an interest rate. This is the rate which induces an investor to defer his or her current preference of money over future preference. This rate is called the Rate of Time Preference. For example, if a Lisa's rate of time preference is 5%, she will agree to forego the opportunity of receiving $100 today, if she is offered $105 after one year. This rate corresponds with the market interest rate and depends on the consumer's expectations of the future income and other factors.

If the future income is expected to be higher than the consumer's current income, he or she will have a high rate of time preference; thus, the interest rate has to be high enough to induce savings instead of spending. Similarly, if the future income is expected to be less than the current income, a rational consumer will be inclined to save even if the interest rate is low. Also, the consumer's rate of time preference (hence the interest rate demanded) is likely to rise as the amount of his or her savings rises. Therefore, the consumer will limit his or her savings to the amount at which the rate of time preference equals the rate of interest.

Rate of time preference for money is dependent on:

- Current interest rate
- Expected future income
- Amount of current savings

A high time preference is because of substantial focus on the well-being in the present and the immediate future.

Most companies and individuals want to receive money immediately rather than waiting for it for long periods of time. A high time preference is because of substantial focus on the well-being in the present and the immediate future.

There are three main reasons for the time preference for money:

- **Risk:** There is uncertainty about the receipt of money in future. This is a major risk for investors.
- **Preference for present consumption:** Time value of money is because of an individual's or a company's preference for present consumption because of need or urgency.
- **Investment opportunities:** Most individuals and companies have a preference for present money because of the availability of opportunities of investment for earning additional cash flow. For example, an individual who is offered $10,000 now or a year later would prefer $10,000 now as it can be invested and can earn interest.

Ex 2.2: Future Value

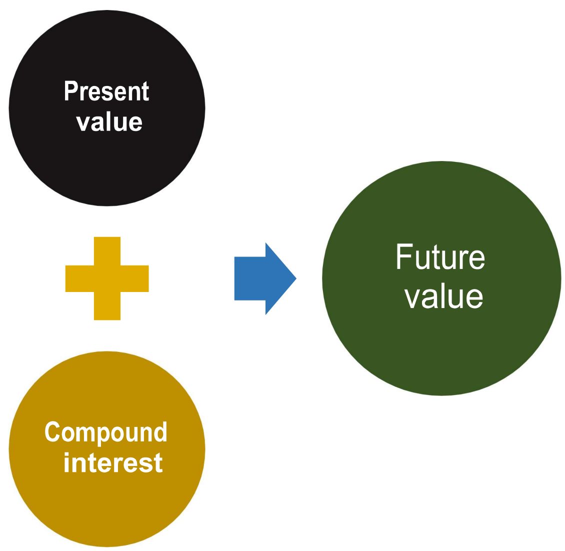
Future value (FV) of a present single sum of money is the amount that will be obtained in future if the present single sum of money is invested on a given date at a given rate of interest.

The future balance, called the FV, can over a long period of time, grow to be a significant amount for two reasons:

- The initial deposit earns interest, and
- The interest added to your account will also earn interest

Earning interest on the previously earned interest is known as compound interest. The FV is the sum of the present value and the compound interest.

The calculation of FV determines just how much a single deposit, investment, or balance will grow to, assuming it is left untouched and earns compound interest at a specified interest rate. The calculation of the FV of a single amount can also be used to predict what a present cost of an item will grow to at a future date, when the item's cost increases at a constant rate. Additionally, the formula for computing the FV can be used to determine either the interest rate or the length of time necessary to reach a desired FV.



The calculation of FV depends on:

- Determines the value of a single deposit, investment, or balance
- Predicts the future cost of an item
- Determines either the interest rate or the length of time necessary to reach a desired FV

Formula

The FV of a single sum of money is calculated by using the following formula:

FV = Present value (PV) × (1 + i)n

where,

I is the interest rate per compounding period

n is the number of compounding periods

Example: 01

An amount of $10,000 was invested on Jan 1, 2011, at an annual interest rate of 8%. Calculate the value of the investment on Dec 31, 2013. Compounding is done on a quarterly basis.

Solution:

Present value (PV) = $10,000

Compounding periods (n) = 3 × 4 = 12

Interest rate (i) = 8%/4 = 2%

Future value (FV) = $10,000 × (1 + 2%)12 = $10,000 × 1.0212 ≈ $10,000 × 1.268242 ≈ $12,682.42

The FV of a single amount can also be calculated easily in Excel, using the formula FV.

	A	B	C
1	Future Value of a Single Amount		
2	Present value (PV)		$10,000.00
3	Period (NPER)	3*4	12
4	Rate	8%/4	2.00%
5	FV	FV(C4,C3,0,-C2,0)	$12,682.42

Example: 02

If you invest Rs. 5000 today at a compound interest of 9%, what will be its FV after 25 years?

Solution:

PV	5,000.00
Rate	9%
NPER	25
FV	**Rs. 43,115.40**

Hint: =FV(C4,C5,0,-C3)

[=fv(rate, time, pmt, -pv, 0)

Ex 2.3: Simple Interest and Compound Interest

Interest is the payment of extra money made by a borrower for having used the money for a certain period of time.

For example, if Sam borrows $100 from John for 1 year, he needs to pay back $100 plus some extra amount for having used John's money. The extra payment made is known as interest. The money actually borrowed is called principal and the total sum consisting of the principal and the interest is called the amount. Note that the interest is always calculated for a certain period at a specified rate.

Simple interest

It is always calculated on the principal amount only. It is assumed that simple interest is paid at the end of a specified period regularly. Even if the interest is not paid regularly, it does not get accumulated for the calculation of further interest.

For instance, suppose $100 is borrowed at 10% p.a. Then, at the end of the first year, the simple interest works out to $10. Even if it is not actually paid, the interest for the next year is computed only on the original principal borrowed. The accrued interest is never added to the principal for further calculation of interest. Therefore, the simple interest for 5 years works out to $50, i.e., $10×5.

The formula for calculating simple interest, **SI = P * n * i**

where,

P = Principal

n = Number of years (or the period)

i = Rate of interest per unit

Example: 03

Karan deposited Rs. 50,000 in bank for 6 years for Simple interest rate of 8%p.a. calculate the simple interest and total amount that Karan receives at the end of 8^{th} year.

Solution:

Simple Interest = P x T x R

= 50,000 X 6 X 8%

= 24,000

Total Amount = Simple interest + Principal

= 24,000 + 50,000

= 74,000

Compound interest

Compound interest is the interest calculated on the principal and accrued interest. The unpaid interest is added to the principal for computing the interest for the next period.

In other words, compound interest is the interest on the growing principal, and it can be computed annually, half yearly, quarterly, or monthly. Both the principal and the compound interest change from one period to another since the principal changes. This is in contrast to simple interest, where the interest is always calculated on the original principal and it is uniformly equal for any period of time.

The one important fact to be borne is the interest earned on the principal is not withdrawn.

For instance, on $1,000, at 10% p.a., the interest for 1 year is $100. For the second year, the principal will be $1,100 (1000 + 100) and the interest for the second year will be $110. The amount of compound interest grows in the successive years since the principal grows.

For compound interest calculation, the three necessary elements are:

- Principal
- Rate per cent
- The period (Time)

The formula for calculating compound interest is: **$CI = P(1 + i)n - P$**

where,

P = Principal

n = Number of years (or the period)

i = Rate of interest per unit

Ex 2.4: Future Value of Annuity

Annuity is a series of equal payments or receipts accruing over a specified number periods. The equal payments may be yearly, half yearly, quarterly, or any other period. These periodic payments accumulate at compound interest until the total sum becomes due.

Schemes for annuity are offered by banks, insurance companies, and other commercial establishments. Banks promise to pay a definite amount every month throughout the individual's lifetime if he or she deposits a lump sum in the beginning. In case of instalments and hire-purchase agreements, the fixation of each instalment associated with interest is done based on the annuity calculation.

The following terms are associated with annuities:

- **Annuity certain:** When an annuity is payable unconditionally for a given length of time, it is called annuity certain or life annuity.
- **Immediate annuity:** If each of the annuity payment is made at the end of each period, it is called immediate annuity.
- **Annuity due:** If each of the annuity payment is made at the beginning of each period, it is known as annuity due.
- **Deferred annuity:** The annuity where the cash flow starts after a gap of more than one time interval is called deferred annuity. An example is a deposit of $10,000 in a bank for which the bank provides a regular pay back from the second year onwards (i.e., no payment is made in the first year). A good example of deferred annuity are the pension schemes, where a money is deposited with the insurer and a regular flow of money is received after attaining a certain age.
- **Perpetuity and deferred perpetuity:** An annuity where the payment is to continue forever is called perpetuity. However, if the payment of annuity is to commence after a certain period, it is known as deferred perpetuity.

Annuity can be calculated as $FVA_n = A\ [(1 + r)^{n - 1}]\ /\ r$

where,

FVAn = FV of an annuity

n = Time duration of the annuity

A = Constant periodic flow

r = Interest rate

Example: 04

	A	B	C
1	**Future Value of an Annuity**		
2	Present value (PV)		**0**
3	Payment (PMT)		**$1,000.00**
4	Interest rate (RATE)		**10.00%**
5	Years (NPER)		**5**
6	Future value (FV)	FV(C4,5,-C3,0,0)	**$6,105**

Ex 2.5: Future Value Sinking Fund

A sinking fund is a fund set up by accumulating a fixed amount every year to meet any future obligation.

A sinking fund is often set up by a company to set aside money over time, in order to retire its preferred stock, bonds, or debentures. In the case of bonds, incremental payments into the sinking fund can soften the financial impact at maturity. Investors prefer bonds and debentures backed by sinking funds because there is less risk of a default. Sinking Fund can be calculated using the Excel formula PMT.

Example: Continuing the example on the previous slide, suppose $6,105 is to be accumulated at the end of 5 years. Then, how much should be deposited each year at an interest of 10% p.a., so that it grows to $6,105 at the end of the fifth year?

The calculation can be made in Excel, using the PMT formula.

	A	B	C
1	**Sinking Fund**		
2	Present value (PV)		**0**
3	Future value (FV)		**$6,105.00**
4	Years (NPER)		**5**
5	Interest rate (RATE)		**10.00%**
6	Annuity (PMT)	PMT(C5,C4,C2,-C3,0)	**$1,000**

Ex 2.6: Present Value of a Single Cash Flow

Present value (PV) of a future cash flow (inflow or outflow) is the amount of current cash that is of equivalent value to the decision-maker.

If you received $100 today and deposited it into a savings account, it would grow over time to be worth *more* than $100. This fact of financial life is a result of the time value of money, a concept according to which, it's more valuable to receive $100 *now* rather than a year from now. To put it another way, the *present value* of receiving $100 one year from now is less than receiving $100 now.

Accountants use PV calculations to account for the time value of money in different applications.

- Providing service now and getting paid later: Suppose XYZ Inc. provides a service in December 2011 and agrees to be paid $100 in December 2012. According to the time value of money, a part of the $100 is the interest earned for waiting one year for the payment of $100. This means that probably only $91 of the $100 is service revenue earned in 2011 and $9 is the interest that will be earned in 2012. The calculation of PV will remove the interest, so that the amount of the service revenue can be determined.

- Buying land: Another example involves interest calculations in the purchase of land. Suppose a land owner will either sell it for $160,000 today or for $200,000 if you pay at the end of two years. To help analyze the alternatives, a PV calculation can help you to determine the interest rate *implicit* in the second option.
- Repayment of money lent: When a friend promises to return the money borrowed, say $1,000, three years hence. What is the PV of this amount if the interest rate is 10%? The PV can be calculated by discounting $ 1,000 to the present point of time.

Discounting is the process of determining the PV of a series of future cash flows. The process of discounting is simply the inverse of compounding. The PV formula can be readily obtained by modifying the compounding formula:

$$\mathbf{FV_n = PV\ (1+}\boldsymbol{r}\mathbf{)^{n}}$$

The factor $1/(1+r)^{n}$ is called the discounting factor or the PV interest factor ($PVIF_{r,n}$).

The interest rate used for discounting cash flows is also called the discount rate.

The PV of a single cash flow can be calculated with the formula PMT in Excel.

Example: 05

What is the PV of $1,000 receivable 6 years hence since the rate of discount is 10%?

The calculation is done in Excel, as shown:

	A	B	C
1	**Present Value of a Single Cash Flow**		
2	Future value (FV)		$1,000.00
3	Years (NPER)		6
4	Interest rate (RATE)		10.00%
5	Present value (PV)	PV(C4,C3,0,-C2,0)	$564

Ex 2.7: Present Value of Annuity

The PV of an annuity is the current value of a set of cash flows in the future, given a specified rate of return or discount rate. The future cash flows of the annuity are discounted at the discount rate, and the higher the discount rate, the lower the present value of the annuity.

The present value of annuity formula determines the value of a series of future periodic payments at a given time. The present value of annuity formula relies on the concept of time value of money, in that one dollar present day is worth more than that same dollar at a future date.

As with any financial formula that involves a rate, it is important to make sure that the rate is consistent with the other variables in the formula. If the payment is per month, then the rate needs to be per month, and similarly, the rate would need to be the annual rate if the payment is annual. An example would be an annuity that has a 12% annual rate and payments are made monthly. The monthly rate of 1% would need to be used in the formula. Tables of the present value of an ordinary annuity interest factors (PVIFA) are available to simplify computations.

Suppose Michael expects to receive $ 1, 000 annually for 3 years, each receipt occurring at the end of the year. What is the present value of this stream of benefits if the discount rate is 10 percent? The present value of this annuity is simply the sum of the present values of all the inflows of this annuity:

$\$ 1,000[\frac{1}{1.10}] + \$ 1,000 [\frac{1}{1.10}]^2 + \$ 1,000 [\frac{1}{1.10}]^3$

= $ 1, 000 x 0.9091 + $ 1, 000 x 0.8264 + $ 1, 000 x 0.7513

= $ 2,486.8

Objectives:

- The PV of an annuity is the current value of a set of cash flows in the future, given a specified rate of return or discount rate.
- The future cash flows of the annuity are discounted at the discount rate.
- The higher the discount rate, the lower the present value of the annuity.
- The present value of annuity formula determines the value of a series of future periodic payments at a given time.
- Tables of the PV of ordinary annuity interest factors (PVIFA) are available to simplify computations.

Example: 06

	A	B	C
1	**Present Value of an Annuity**		
2	Payment (PMT)		**$1,000.00**
3	Interest rate (RATE)		**10.00%**
4	Years (NPER)		**3**
5	Present value (PV)	PV(C3,C4,-C2,0,0)	**$2,487**

Ex 2.8: Capital Recovery and Loan Amortization

- The Capital Recovery Period (CRP) technique provides a yardstick for estimating the period over which the project's investment will be recovered. In the CRP: Discounted benefits = Investment cost
- The capital recovery factor (CRF) shows the amount of income that can be generated over a period of time, when a lump sum amount is invested.

This equal periodic withdrawal can be calculated easily with the Excel formula PMT.

By taking into account the time value of money, the Capital Recovery Period (CRP) technique provides a yardstick for estimating the period over which the project's investment will be recouped. The quicker this return, the greater the preference for a project. The CRP is the period over which the discounted benefits are equivalent to the investment cost.

The capital recovery factor (CRF) shows the amount of income that can be generated over a period of time, when a lump sum amount is invested.

In other words, the CRF shows how much can be withdrawn in equal amounts at the end of each of n years if $1 is deposited initially at i percent.

Example: 07

Suppose you want to invest $1,000 today, for a period of 4 years. If the interest rate is 10%, how much income per year should you receive, to recover your investment?

Solution:

	A	B	C
1	**Capital Recovery**		
2	Present value (PV)		$10,000.00
3	Future value (FV)		0
4	Years (NPER)		4
5	Interest rate (RATE)		10.00%
6	Annuity (PMT)	PMT(C5,C4,-C2,0,0)	$3,155

One of the most familiar applications of time value of money concepts is the amortization schedule.

In an amortizing loan, the principal is paid down over the life of the loan (that is, amortized) according to an amortization schedule, typically through equal payments. An amortization schedule is a list of balances, payments, and interest charges from loan inception until payoff.

This equal periodic payments can be calculated easily using the Excel formula PMT

Year	Beginning Balance	Payment	Interest Paid	Principal Paid	Ending Balance
1	$10,000	$3,087	$900.00	$2,187	$7,813
2	$7,813	$3,087	$703.20	$2,383	$5,430
3	$5,430	$3,087	$488.68	$2,598	$2,832
4	$2,832	$3,087	$254.86	$2,832	$0

Ex 2.9: PV of Perpetuity

Perpetuity is an infinite series of periodic payments of equal face value. In other words, perpetuity is a situation where a constant payment is to be made periodically for an infinite amount of time. It as an annuity having no end and that is why the perpetuity is sometimes called perpetual annuity.

Although the total face value of perpetuity is infinite and undeterminable, its present value is not. According to the time value of money principle, the present value of perpetuity is the sum of the discounted value of each periodic payment of the perpetuity. Present value of perpetuity is finite because the discounted value of far future payments of the perpetuity reduces considerably and reaches close to zero.

Formula

The following formula is used to calculate the present value of perpetuity:

PV of perpetuity = A/ r

where,

A is the fixed periodic payment

r is the interest rate or discount rate per compounding period

Example: 08

Calculate the present value on Jan 1, 2010, of a perpetuity paying \$1,000 at the end of each month, starting from January 2010. The monthly discount rate is 0.8%.

Solution:

Periodic payment A = \$1,000

Discount rate i = 0.8%

PV = \$1,000 ÷ 0.8% = \$125,000

Ex 2.10: PV of an Uneven Cash Flow

The formula for the PV of an annuity assumes equal cash flows at each time period. However, sometimes, cash flows are not even.

An annuity is an asset that will pay equal amounts of money at regular time periods over its life. Therefore, an annuity can be thought of as a security with equal expected cash flows usually paid annually, semi-annually, quarterly, or monthly. The payment of dividends or payments from a lawsuit settlement are typical annuities. However, expected future cash flows from a security with the uncertainty of market and economic conditions rarely follow such a regular schedule.

Investors often calculate the value of an asset such as stocks, bonds, and options by evaluating the expected future cash flows the asset will bring to its owner.

For example, suppose a corporation pays regular dividends to its stock holders each quarter of the year. These four payments represent cash flows to the stock holders for each share of stock owned at the time of the dividend payment. However, dividend payments made in the past are no guarantee of payments in the future. Furthermore, even corporate policies that guarantee dividend payments cannot be enforced if a company goes bankrupt and is forced to default on its promise to pay.

Suppose that an investor has determined that the expected future cash flows of an asset will follow the following schedule:

- Year 1: \$1,000
- Year 2: \$1,500
- Year 3: \$800
- Year 4: \$1,100
- Year 5: \$ 400

Although the payment of these cash flows is quite regular, the amounts differ from one time period to another. The standard annuity formula for determining the present value of this asset is insufficient because it assumes that payments are equal. Calculating the present value of these cash flows is a bit more complicated.

Suppose the discount rate from the cash flows above is 10%. This means that a required return of 10% is necessary to make the investment worthwhile. What is the PV of the cash flows at 10%?

To calculate the PV of an uneven cash flow, each cash flow must be considered to be a component of the total PV of the asset. Then, adding up the components gives the total PV. However, this calculation can be done easily with the help of the Excel formula NPV as shown.

Example: 09

	A	B	C
1	Present Value of Flow		
2	Year		Cash Flows
3	1		$1,000
4	2		$1,500
5	3		$800
6	4		$1,100
7	5		$400
8	Rate		10%
9	Present value	NPV(C8,C3,C4,C5,C6,C7)	$3,749

Example: 10

	A	B	C	D	E	F	G
1	Present Value of an Uneven Cash Flow						
2		Year	0	1	2	3	4
3		Cash flow	$5,000.00	$6,000.00	$8,000.00	$9,000.00	$8,000.00
4		Rate	14%				

Solution:

Year	0	1	2	3	4
Cash flow	$5,000.00	$6,000.00	$8,000.00	$9,000.00	$8,000.00
Rate	14%				
NPV	$23,886.21				

Formula =npv(14%, cash flow)

Ex 2.11: PV of Growing Annuity

The PV of a growing annuity formula calculates the present day value of a series of future periodic payments that grow at a proportionate rate. A growing annuity may sometimes be referred to as an increasing annuity.

A simple example of a growing annuity would be an individual who receives $100 the first year and successive payments increase by 10% per year for a total of 3 years. This would be a receipt of $100, $110, and $121, respectively.

The PV of a growing annuity formula relies on the concept of time value of money. The premise to this concept is that a specific quantity of money is worth more today than at a future time.

Like all financial formulas that involve a rate, it is important to correlate the rate per period to the number of periods in the PV of a growing annuity formula. If the payments are monthly, then the rate would need to be the monthly rate.

The calculation of the PV of a growing value can be done with the Excel formula NPV, just like the calculation of PV of an uneven cash flow.

Objectives:

- A growing annuity is a finite number of cash flows growing at a constant rate.
- The PV of a growing annuity formula calculates the present day value of a series of future periodic payments that grow at a proportionate rate.
- It is important to correlate the rate per period to the number of periods in the PV of a growing annuity formula. If the payments are monthly, then the rate would need to be the monthly rate.
- The PV of a growing annuity can be calculated using the Excel formula NPV, just like the calculation of PV of an uneven cash flow.

Summary

In this chapter, you learned that:

- The preference for money now, as compared to future money, is called time preference for money.
- FV of money is the amount to be obtained in future, when any money is invested for a period and accrues interest. The FV can be calculated using the formula FV in Excel.
- Simple interest is calculated on the principal amount only and not on the accumulated amount.
- Compound interest is calculated on the principal amount and the accumulated amount interest.
- Annuity is a series of equal payments or receipts accruing over a specified number periods. The FV of an annuity can be calculated using the Excel formula FV.
- Sinking fund is a fund set up by accumulating a fixed amount every year to meet any future obligation. It can be calculated using the Excel formula PMT.
- The PV of a future cash flow (inflow or outflow) is the amount of current cash that is of equivalent value to the decision-maker. The PV of a future cash flow can be calculated using the Excel formula PMT.
- Discounting is the process of determining the PV of a series of future cash flows.
- The PV of an annuity is sum of the present value of a series of equal periodic payments. The PV of an annuity can be calculated easily using the Excel formula PV.
- CRF shows the amount of income that can be generated over a period of time, when a lump sum amount is invested.
- An amortization schedule is a list of balances, payments, and interest charges from loan inception until payoff.
- A perpetuity is an annuity with infinite duration. The PV of a perpetuity is equal to the fixed periodic payment divided by the interest rate.
- A growing annuity is a finite number of cash flows growing at a constant rate. The PV of a growing annuity can be calculated using the Excel formula NPV.

Questions Based on this Chapter

1. To find the PV of a sum of $10,000 to be received at the end of each year for the next 5 years at a rate of 10%, which of the following value is used?
   a. PV of a single cash flow table
   b. PV of annuity table
   c. FV of a single cash flow table
   d. FV of annuity table
2. Given an investment of $10,000 for a period of 1 year, which of the following schemes provides the maximum returns?
   a. 12% interest compounded annually
   b. 12% interest compounded quarterly
   c. 12% interest compounded monthly
   d. 12% interest compounded daily
3. Which Excel formula is used to calculate the PV of an uneven cash flow?
4. What is Sinking fund and when is it set up by a company?
5. How can you calculate the Present value of Perpetuity?
6. Write down the different types of perpertuity?
7. Calculate Future Value.

Present Value (PV)	0
Payment (PMT)	8000
Interest Rate	12%
Years (NPER)	8
Future Value (FV)	**?**

8. Calculate PMT.

Present Value (PV)	0
Interest Rate	10%
Years (NPER)	12
Future Value (FV)	100000
Payment (PMT)	**?**

9. Calculate Future Value of Compounding factor.

Present Value(PV)	100000
Period (Years)	15
Rate (Compound)	12%
Future Value (compounding factor)	**?**

10. Calculate NPV.

Year	Cashflow
1	2500
2	5500
3	7500
4	4540
5	5750
Discounting Rate	12%
NPV	**?**

11. Rahul took a loan of Rs.5,00,000 for 10 years. He needs to pay interest, which contains rate of 12.5%. Find out the yearly EMI Rahul needs to pay.

Chapter 3: Bond and Equity Valuation